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LETTER TO THE EDITOR

The statistics of eigenvector components of random band matrices: analytical results

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Abstract. By using a supersymmetric formalism we calculate analytically all higher moments $P_q = \overline{\sum_n |\Psi_n|^{2q}}$ generalizing the inverse participation ratio P_2 where $\Psi_n (1 \leqslant n \leqslant N)$ stands for the nth component of an eigenvector of a large random matrix with a band structure. On this basis we reconstruct the whole probability distribution function of eigenvector components. The relation with known numerical results is discussed.

Large $N \times N$ random matrices whose elements differ appreciably from zero within a band of a typical size b around the main diagonal has recently attracted a lot of research interest [1–12]. The property that makes this class of random matrices interesting is the localization of their eigenvectors [2], which is in contrast to the situation in the usually studied Gaussian orthogonal, unitary and symplectic ensembles (GOE, GUE, GSE). Such a qualitatively different structure of eigenvectors affects the eigenvalues statistics which changes between a Poissonic and a Wigner–Dyson one depending on the only scaling parameter $x = b^2/N$ [3]. This interpolation property was an original motivation for the introduction of the ensemble of random-band matrices (RBM) [1] as an attempt to describe the intermediate level statistics typical for Hamiltonian systems in a transition regime between the complete integrability (corresponding to Poissonic statistics [13]) and fully developed chaos characterized by the Wigner–Dyson statistics [14].

Considerable interest in the ensemble of RBM was stimulated by investigation of quantum behaviour of periodically driven Hamiltonian systems. A paradigmatic system in this class is a so-called kicked rotator (KR) (see review [15]). Classically it exhibits an unbounded diffusion in the angular momentum space when a strength of kicks exceeds some critical value. It was observed, however, that in a quasiclassical regime quantum effects suppress a classical diffusion [16] in close analogy with the Anderson localization of a quantum particle by a random potential. A formal connection with a kind of 1D tight-binding model has been found [17] that revived a general interest to a localization in one-dimensional systems. This effect of the 'dynamical localization' was claimed to be experimentally observed in ionization experiments in a monochromatic field (see review [18]).

In an appropriate basis the matrix of the evolution operator $\hat{U}$ for the KR that relates values of the wavefunction in one period of perturbation appears to have a band structure with pseudorandom elements within the band [15]. The width of the band proves to be large in the quasiclassic regime. All these observations gave a boost to the investigations

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of statistical properties of eigenvectors and eigenvalues of RBM in order to use the extracted information for understanding of related properties of KR.

Intensive numerical simulations [2–7, 9, 10, 12] revealed some universality in the statistical properties of RBM. In particular, it was shown that the whole distribution function of eigenvector components is dependent on the same scaling parameter x^* as the probability distribution function of eigenvalue spacings [9]. The origin of such a scaling behaviour was explained in our previous paper [8] where it was demonstrated that the localization length ξ of an eigenvector is proportional to b^2 when the matrix size N tends to infinity. Exploiting the analogy with the theory of disordered conductors we therefore conclude that all statistical properties of the system should be dependent only on the ratio $\xi/N \propto x^*$ which is nothing but the requirement of the well known scaling hypothesis [19].

It is worth mentioning that RBM-like ensembles also arise in solid state physics in the course of investigation of conductance fluctuations of quasi-one-dimensional disordered systems [20]. Namely, an isolated thick wire with length L and a cross-section S ($k_F^2 S \gg 1, k_F l \sim 1$ where k_F is the Fermi wavenumber and l is the electron mean free path) corresponds to the RBM with a bandwidth $b \propto k_F^2 S$ and a matrix size $N \propto k_F^3 LS$. Let us also mention that RBM were claimed to be relevant for the explanation of the behaviour of mesoscopic cylinders threaded by a linearly time-dependent magnetic flux [21].

In order to describe statistics of eigenvectors of RBM in a quantitative way a set of generalized localization lengths ξ_q was introduced [2, 4] according to the following definition:

$$\begin{aligned} \xi_q &= P_q^{1/(1-q)} \quad : \quad P_q = \sum_n \overline{|\Psi_n^{(k)}|^{2q}} \qquad q \geqslant 2 \\ \ln \xi_1 &= -\sum_n \overline{|\Psi_n^{(k)}|^2 \ln |\Psi_n^{(k)}|^2} \end{aligned} \tag{1}$$

where $\Psi_n^{(k)}$ is the nth component of the kth eigenvector of the matrix and the bar means both the averaging over the disorder and over a set of eigenvectors corresponding to a narrow window of eigenvalues E_k at a given point of spectrum. Quantities P_q are a natural generalization of the inverse participation ratio P_2 which has a simple physical meaning of probability for a quantum particle with RBM-type Hamiltonian to return to the initial position after infinite time.

In [2] it was noticed that in a wide range of values of $x^* (x^* \lesssim 10)$ the following scaling relation for the entropic localization length ξ_1 holds numerically:

$$\frac{\beta_1}{1-\beta_1} = C_1 x^* \qquad \beta_1 \equiv \xi_1/N\,. \tag{2}$$

In the related publication [4], it was claimed that the relation analogous to (2) is also true for all $\beta_q = \xi_q/\xi_q^{\mathrm{GOE}}$ where ξ_q^{GOE} is the generalized localization length for GOE.

In our preceding publication [11] we calculated analytically the inverse participation ratio P_2 and demonstrated that the scaling law (2) holds exactly in this case. In the present paper we generalize our method in order to calculate higher moments P_q, $q = 3, 4 \ldots$. It allows us to derive a closed analytical expression for the whole distribution function of $|\Psi_n^{(k)}|^2$.

We consider the ensemble of random bandlike $N \times N$ ($N \gg 1$) matrices $\hat{H}$—real symmetric or Hermitian—whose elements $H_{ij}^* = H_{ij}$ are distributed independently according to the Gaussian law with mean zero value and variances $\langle H_{ij}^* H_{ji} \rangle = \frac{1}{2} a(|i -$

$j|)(1+\delta_{ij})$. The function $a(r)$ decays exponentially for r exceeding the bandwidth $b \gg 1$. For the sake of simplicity we consider the case of Hermitian RBM in detail presenting the final results for real RBM at the end of the letter. Let us note that physically the ensemble of Hermitean (real) matrices corresponds to a quantum system with broken (unbroken) time-reversal invariance.

Our aim is to calculate all moments of the eigenvector (1) that can be written in the form

$$P_q \equiv N^{1-q} \int_0^\infty \mathrm{d}y\, \mathcal{P}(y) y^q = \frac{1}{N} \sum_{n=1}^{N} P_q(E,n)$$
$$P_q(E,n) = \frac{1}{\rho(E)} \overline{\sum_k |\Psi_n^{(k)}|^{2q} \delta(E-E_k)} \tag{3}$$

where $\rho(E)$ is the density of states, averaging is performed over the matrix ensemble and $\mathcal{P}(y)$ is a distribution function of the variable $y_n = N|\Psi_n|^2$. For this purpose we introduce the following set of correlation function:

$$K_{l,m}(n,\eta) = \langle n|(E+\mathrm{i}\eta - \hat{H})^{-1}|n\rangle^l \langle n|(E-\mathrm{i}\eta-\hat{H})^{-1}|n\rangle^m$$
$$l, m \geqslant 1 \quad \eta > 0 \,. \tag{4}$$

In the limit $\eta \to 0$ they have a singularity of the form η^{1-l-m} which can be easily extracted, that gives

$$P_q(E,n) = \frac{\mathrm{i}^{l-m}}{2\pi\rho(E)} \frac{(l-1)!(m-1)!}{(l+m-2)!} \lim_{\eta\to 0} (2\eta)^{l+m-1} \overline{K_{l,m}(n,\eta)} \qquad q = l+m \,. \tag{5}$$

To calculate the correlators (4) we use the supersymmetric formalism [22]. Here we give only the sketch of the procedure [8, 11, 22]. The necessary steps are:

(i) to express the correlation function (4) in terms of the integral over superfields:

$$K_{l,m}(n,\eta) = \frac{\mathrm{i}^{l-m}}{l!m!} \int \prod_{i=1}^{N} \mathrm{d}\Phi_i\, \mathrm{d}\Phi_i^{+} (\phi_{1,n}^{*}\phi_{1,n})^l (\phi_{2,n}^{*}\phi_{2,n})^m$$
$$\times \exp\left\{ \mathrm{i} \sum_{i,j=1}^{N} \Phi_i^{+} L^{1/2} ((E+\mathrm{i}\eta L)\delta_{ij} - H_{ij}) L^{1/2} \Phi_j \right\} \tag{6}$$

where a supervector Φ_i^{+} has the structure $\Phi_i^{+} = (\phi_{1,i}^{*}, \chi_{1,i}^{*}; \phi_{2,i}^{*}\chi_{2,i}^{*})$ with two commuting variables ϕ and two Grassmannian variables χ; $L = \mathrm{diag}\{1, 1, -1, -1\}$;

(ii) to average (6) over the disorder and to decouple the resulting quartic term in the exponent introducing a supermatrix composite variable Q (the Hubbard–Stratonovich transformation);

(iii) to perform Gaussian integration over supervectors Φ_i and to make use of a saddle-point approximation for the remaining integral over Q that is justified in the limit $b^2, N \gg 1$. As a result we obtain

$$\overline{K_{l,m}(n,\eta)} = \int \prod_{n=1}^{N} \mathrm{d}\mu(Q(n)) F[Q(n)] \exp\{-S[Q]\}$$
$$F[Q] = \sum_j \frac{l!}{(l-j)!j!} \frac{m!}{(m-j)!j!} Q_{11,\mathrm{bb}}^{l-j}(n) Q_{22,\mathrm{bb}}^{m-j}(n) Q_{12,\mathrm{bb}}^{j}(n) Q_{21,\mathrm{bb}}^{j}(n) \,. \tag{7}$$

Here $Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$ is 4×4 supermatrix belonging to the graded co-set space $U(1, \frac{1}{2})$ (explicit parametrization of Q can be found in [23]); all $Q_{pp'}$ are 2×2 supermatrices and subscript 'bb' denotes their boson–boson components. The action $S[Q]$ in (7) is given by

$$S[Q] = \sum_i \mathrm{Str}\{-\gamma Q_i Q_{i+1} + \mathrm{i}\epsilon Q_i L\} \tag{8}$$

where

$$\begin{aligned} &\gamma = (\pi\rho)^2 B_2/4 \qquad \epsilon = \pi\rho\eta/2 \\ &\pi\rho = (2B_0 - E^2)^{1/2}/B_0 \qquad B_k = \sum_{n=-\infty}^{\infty} n^k a(n) \end{aligned} \tag{9}$$

and Str stands for the supertrace [22].

In view of a one-dimensional structure of (8) the correlation function (7) can be expressed in the form

$$\overline{K_{l,m}(n,\eta)} = \int \mathrm{d}\mu(Q) F(Q) Y(Q, N-n) Y(Q, n) \exp\{-\mathrm{i}\epsilon \mathrm{Str}\, QL\} \tag{10}$$

where $Y(Q, n)$ satisfies the reccurence equation

$$\begin{aligned} &Y(Q', n+1) = \int \mathrm{d}\mu(Q) Y(Q, n) L(Q, Q') \qquad Y(Q, 0) = 1 \\ &L(Q, Q') = \exp\{\gamma \mathrm{Str}\, QQ' - \mathrm{i}\epsilon \mathrm{Str}\, QL\}. \end{aligned} \tag{11}$$

Making use of the Efetov parametrization of matrix Q [22, 23] it is possible to show that $Y(Q, n)$ depends only on 'eigenvalues' λ_1, λ_2 of the Q_{12} block. The integration over remaining degrees of freedom can be carried out using the general method developed in [22, 23].

In the limit $\epsilon \to 0$ that we are interested in, the main contribution to the integral (10) comes from the region $\lambda_1 \sim \epsilon^{-1}$ and the function $Y(Q, n)$ proves to be dependent only on the variable $z = \lambda_1\epsilon$. Moreover, in this asymptotic domain

$$Q_{11,\mathrm{bb}} Q_{22,\mathrm{bb}} \approx Q_{12,\mathrm{bb}} Q_{21,\mathrm{bb}}. \tag{12}$$

Performing the computation, we get

$$P_q^{\mathrm{H}}(E, n) = q(q-1)(\pi\rho)^{q-1} \int_0^\infty \mathrm{d}z\, z^{q-2} \exp\{-\pi\rho z\} Y(z, n) Y(z, N-n). \tag{13}$$

We put the superscript H in order to remind the reader that the derivation was performed for Hermitian matrices.

The recurrence relation (11) in the limit $N, b^2 \gg 1$ may be reduced to the following differential equation [11]†:

$$\frac{\partial Y(y,\tau)}{\partial \tau} = \left(-y + y^2 \frac{\partial^2}{\partial y^2}\right) Y(y,\tau) \qquad Y(y, 0) = 1 \tag{14}$$

† Let us note that the same equation (14) appeared in the course of investigation of a strictly one-dimensional system by the Berezinskii technique [24, 25].

where $y = 4\gamma\pi\rho z$ and we introduced the continuous variable $\tau = n/4\gamma$ instead of the discrete index n. Then, averaging (13) over n we get:

$$P_q^{\mathrm{H}}(E) = \frac{q(q-1)}{(4\gamma)^{q-1}} \int_0^\infty \mathrm{d}y\, y^{q-2} \int_0^1 \mathrm{d}v\, Y(y, x(1-v))Y(y, xv) \tag{15}$$

where the parameter $x = N/4\gamma \propto N/b^2$ is the only scaling variable characterizing the RBM ensemble.

The solution $Y(y, \tau)$ of (15) can be found [11, 25] in the form of the Lebedev–Kantorovich expansion:

$$Y(y,\tau) = 2y^{1/2}\left[K_1(2\sqrt{y}) + \frac{2}{\pi}\int_0^\infty \mathrm{d}\nu\, \frac{\nu}{1+\nu^2}\sinh\{\pi\nu/2\} \times K_{\mathrm{i}\nu}(2\sqrt{y})\exp\left\{-\frac{1+\nu^2}{4}\tau\right\}\right]. \tag{16}$$

By using expressions (15) for P_q^{H} we can restore the distribution function $\mathcal{P}_x^{\mathrm{H}}(y)$ introduced in (3) in terms of the function $Y(y, \tau)$:

$$\mathcal{P}_x^{\mathrm{H}}(y) = \frac{1}{x^2}\frac{\partial^2}{\partial y^2}\int_0^1 \mathrm{d}v\, Y(y; x(1-v))Y(y; xv)\,. \tag{17}$$

Let us first consider two limiting cases $x \gg 1$ and $x \ll 1$. When considering small values of x, that corresponds to the matrix size N much smaller than the localization length $\xi \propto b^2$, it is more convenient to solve (4) iteratively rather than to use the exact expression (16). That gives for $\tau \ll 1$, $\tau y \lesssim 1$

$$Y(y,\tau) \approx \mathrm{e}^{-\tau y}\left\{1 + \tau\tfrac{1}{3}(\tau y)^2 + \tau^2\left[\tfrac{1}{6}(\tau y)^2 - \tfrac{4}{15}(\tau y)^3 + \tfrac{1}{18}(\tau y)^4\right] + \mathrm{O}(\tau^3)\right\}. \tag{18}$$

Substituting (18) into (17) we get the asymptotic expansion for $x \ll 1$:

$$\mathcal{P}_x^{\mathrm{H}}(y) = \mathrm{e}^{-y}\left\{1 + x\left[\tfrac{1}{3} - \tfrac{2}{3}y + \tfrac{1}{6}y^2\right] + x^2\left[\tfrac{2}{15} - \tfrac{4}{5}y + \tfrac{4}{5}y^2 - \tfrac{2}{9}y^3 + \tfrac{1}{60}y^4\right] + \mathrm{O}(x^3)\right\}. \tag{19}$$

The leading term in this expression corresponds to the pure GUE case ($x = 0$); higher terms are due to localization effects.

By using (19) we easily get the corresponding expansion for reduced moments $\tilde{\beta}_q^{\mathrm{H}}(x) = P_q^{\mathrm{H}}(x)/P_q^{\mathrm{GUE}}$:

$$\tilde{\beta}_q^{\mathrm{H}}(x) = 1 + \tfrac{1}{6}q(q-1)x + \tfrac{1}{180}q(q-1)(q-2)(3q-1)x^2 + \mathrm{O}(x^3)\,. \tag{20}$$

In the opposite case $x \gg 1$ corresponding to the strongly localized eigenstates the leading-order expression for $\mathcal{P}_x^{\mathrm{H}}(y)$ can be obtained if we take into account only the first term in (16):

$$\mathcal{P}_x^{\mathrm{H}}(y) \approx \frac{8}{x^2}\left[K_1^2(2\sqrt{y/x}) + K_0^2(2\sqrt{y/x})\right] \qquad x \to \infty \qquad y \gg x\,\mathrm{e}^{-x}\,. \tag{21}$$

The expressions for reduced moments $\tilde{\beta}_q$ can be obtained at $x \gg 1$ with an exponential accuracy:

$$\tilde{\beta}_q^{\mathrm{H}}(x) = x^{q-1}\frac{q![(q-1)!]^2}{(2q-1)!} + A_q\frac{x^{q-2}}{(q-2)!} + \mathrm{O}(\mathrm{e}^{-x/4})$$
$$A_2 = 1 \qquad A_3 = \frac{4}{3} \qquad A_4 = \frac{11}{5} \quad \ldots . \tag{22}$$

Having at our disposal the distribution function in two limiting cases, equations (19) and (21), we are able to calculate the asymptotic expressions for the entropic localization length ξ_1 as well:

$$\xi_1 = N\exp\left\{-\int \mathrm{d}y\, y\ln y\, \mathcal{P}_x^{\mathrm{H}}(y)\right\} \qquad \frac{\xi_1}{\xi_1^{\mathrm{GUE}}} = \begin{cases} x^{-1}\exp\{1+C\} & x \gg 1 \\ 1 - \frac{1}{6}x + \frac{1}{40}x^2 & x \ll 1 \end{cases} \tag{23}$$

where $\xi_1^{\mathrm{GUE}} = N\exp\{C-1\}$ and $C = 0.577$ is the Euler constant.

For $q = 2$ it turns out to be possible to calculate $\tilde{\beta}_2(x)$ exactly [11]: $\tilde{\beta}_2^{\mathrm{H}}(x) = 1 + x/3$. For higher q the value of the reduced moments $\tilde{\beta}_q$ can be obtained by substituting (16) into (15), performing the integration over y and v analytically and calculating the remaining double integral numerically. It is interesting to note that the two asymptotics, (20) and (22), match very well in the intermediate region and, when combined, perfectly describe the function $\tilde{\beta}_q(x)$ at the whole range of x.

So far we discussed the case of RBM with Hermitian structure. The supersymmetric approach allows us to perform all calculation for real symmetric (RS) band matrices as well. The resulting expressions are quite similar to those presented above. Namely, the inverse participation ratio P_q^{RS} is proportional to P_q^{H}:

$$P_q^{\mathrm{RS}}(x) = \frac{(2q-1)!!}{q!}P_q^{\mathrm{H}}(2x) \qquad \tilde{\beta}_q^{\mathrm{RS}}(x) = \tilde{\beta}_q^{\mathrm{H}}(2x) \tag{24}$$

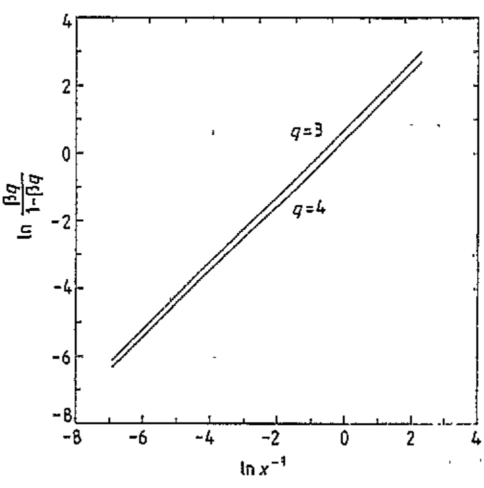

Figure 1. The log–log plot of $\frac{\beta_q}{1-\beta_q}$ versus x^{-1} for $q = 3, 4$.

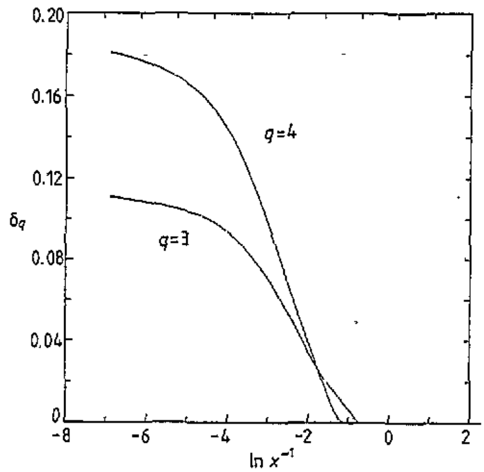

Figure 2. The deviation from the linear law, (28): the plot of $\delta_q(x) = \ln[x\beta_q/(1-\beta q)] - d_q^-$ versus $\ln x^{-1}$ for $q = 3, 4$.

where now $\tilde{\beta}_q^{\mathrm{RS}}(x) = P_q^{\mathrm{RS}}(x)/P_q^{\mathrm{GOE}}$; $P_q^{\mathrm{GOE}} = P_q^{\mathrm{RS}}(0)$. For the distribution function of the components of eigenvector we get from (24)

$$\mathcal{P}_x^{\mathrm{RS}}(y) = \frac{1}{\pi y^{1/2}} \int_{y/2}^{\infty} \mathrm{d}p\,(2p-y)^{-1/2} \mathcal{P}_{2x}^{\mathrm{H}}(p)\,. \tag{25}$$

For $x \ll 1$ it gives the expression

$$\mathcal{P}_x^{\mathrm{RS}}(y) = \frac{\mathrm{e}^{-y/2}}{\sqrt{2\pi y}} \left\{ 1 + x\left(\frac{1}{4} - \frac{y}{2} + \frac{y^2}{12}\right) + x^2\left(\frac{y^4}{240} - \frac{17}{180}y^3 + \frac{13}{24}y^2 - \frac{3}{4}y + \frac{5}{48}\right) + \mathrm{O}(x^3) \right\} \tag{26}$$

and the limiting expression at $x \to \infty$, $y \gg x\,\mathrm{e}^{-x}$ is

$$\mathcal{P}^{\mathrm{RS}}(y)|_{x\to\infty} = \frac{8}{x^2}\sqrt{x/2y}\,K_1\big(2\sqrt{2y/x}\big)\,. \tag{27}$$

In order to compare our results with the empirical scaling law [4]

$$\beta_q/(1-\beta_q) = C_q x^* \tag{28}$$

where $\beta_q = \tilde{\beta}_q^{-1/(q-1)}$ and $x^* = b^2/N \equiv (b^2/\gamma)x^{-1}$, we plotted $\ln[\beta_q/(1-\beta_q)]$ versus $\ln x^{-1}$ (figure 1) for $q = 3, 4$. If the relation (28) were true, the curves would be straight lines. We see that in a wide range of x it is approximately true with very high accuracy. However, we would like to stress that (28) is not an exact relation for $q \neq 2$ that can be easily seen from the asymptotic expressions (20) and (22). In particular, the quantity $d_q = \ln[x\beta_q/(1-\beta_q)]$ has different limiting values $d_q^{\pm}$ at $\ln x \to \pm\infty$:

$$d_q^+ = \frac{1}{q-1} \ln \frac{(2q-1)!}{q!(q-1)!^2} \qquad d_q^- = -\ln q/6\,. \tag{29}$$

It is easy to see that indeed $d_q^+ = d_q^-$ only at $q = 2$ when the relation (28) is exact [11]. For $q \neq 2$ the difference $\Delta_q = d_q^+ - d_q^-$ has a non-zero value but is rather small: $\Delta_1 = -0.215$, $\Delta_3 = 0.111$, $\Delta_4 = 0.183, \ldots$, $\Delta_\infty = 0.594$. This explains the good accuracy of the conjecture (28). Deviations from the linear law (28) are displayed in figure 2 for $q = 3, 4$.

Let us also note that a detailed comparison with numerical results of other papers [2, 7] is difficult since the authors of the cited papers averaged their data over the whole spectrum whereas our results are derived at a given point of the spectrum. Further numerical work is desirable from this point of view.

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